647510 **BSCmaMJ204x-2C** **Seat No :_________**

B.Sc. Semester - 2 (NEP-2020) Examination
MARCH/APRIL-2026 (As Per Syllabus Year June-2023)
MATH: Partial Derivatives & Differential Geometry (Major-4)

Time: 2:00 Hours **Marks: 50**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one. (05)

(1) Define: Homogeneous function. If u is homogeneous function of x and y of degree n then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$.

(2) If $u = log(x^2+y^2) + tan^{-1}\frac{y}{x}$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

Que.1(B) Answer any one. (05)

(1) If $u = f(x,y)$, where $x = e^a cost, y = e^a sint$ then show that
$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial u}{\partial a}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$.

(2) If $f(x,y) = 0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Que.2(A) Answer any one. (05)

(1) Expand $e^x cosy$ in powers of x and y up to degree three.

(2) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Que.2(B) Answer any one. (05)

(1) If $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

(2) Find maxima and minima for the function $f(x,y) = x^2 - 2xy + \frac{y^3}{3} - 3y$.

Que.3(A) Answer any one. (05)

(1) If ρ is the radius of curvature for the polar curve $r = f(\theta)$ then
Prove that: $\rho = \frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2+2{r_1}^2-rr_2}$, where $r_1 = \frac{dr}{d\theta}, r_2 = \frac{d^2r}{d\theta^2}$.

(2) Using Newton's method, find the radius of curvature at origin for the curve
$2x^4 - 5y^4 + 3x^2y + xy - 3y^2 + 8x = 0$.

Que.3(B) Answer any one. (05)

(1) Show that radius of curvature at any point on the cycloid
$x = a(\theta + sin\theta), y = a(1 - cos\theta)$ is $4acos\frac{\theta}{2}$.

(2) Show that the radius of curvature off the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ is $\frac{3a\sqrt{2}}{16}$.

Que.4(A) Answer any one. (05)

(1) Define: Point of inflexion. Find points of inflexion for the curve
$y = 3x^4 - 4x^3 + 1$.

(2) If $y = mx + c$ is an asymptote to the curve $y = f(x)$ then prove that $m = \lim_{x\to\pm\infty}\left(\frac{y}{x}\right)$ and $c = \lim_{x\to\pm\infty}(y - mx)$.

Que.4(B) Answer any one. (05)

(1) Find tangents at the point $(-1,-2)$ on the curve $x^3+2x^2+2xy-y^2+5x-2y=0$ and show that the point $(-1,-2)$ is a cusp.

(2) Find all the asymptotes of the curve $x^3+y^3-3axy=0$.

Que.5(A) Answer any one. (05)

(1) If $u=log(tanx+tany+tanz)$ then prove that $sin2x\frac{\partial u}{\partial x}+sin2y\frac{\partial u}{\partial y}+sin2z\frac{\partial u}{\partial z}=2$.

(2) If there is 1% error in the measurement of half of major and minor axis of an ellipse then evaluate relative error in measure of its area.

Que.5(B) Answer any one. (05)

(1) Find radius of curvature at any point (x,y) on the curve $y^2=4ax$.

(2) Find all asymptotes for the curve $x^3+2x^2y-xy^2-2y^3+xy-y^2-1=0$.

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